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A simple scheme for masses and mixings of quarks and neutrinos

Berthold Stech[1]

Institut für Theoretische Physik
Universität Heidelberg
Philosophenweg 16, D-69120 Heidelberg

Abstract

The mass matrices of charged fermions have a simple structure if expressed in powers of the small parameter $\sigma = (m_c/m_t)^{1/2}$. It is suggested that the mass matrix of the three heavy neutrinos occuring in grand unified theories can be expressed in terms of the same parameter. The requirement that these heavy neutrinos carry different $U(1)$ generation quantum numbers gives rise to an almost unique form for this matrix. By applying the see-saw mechanism, the mass splitting of the two lightest neutrinos comes out to be tiny, favoring the vacuum oscillation solution for solar neutrinos. The mixing matrix is of the bimaximal type but contains also CP violating phases.

[1]e-mail: B.Stech@thphys.uni-heidelberg.de

1 Introduction

It is well known that the masses and mixings of quarks show a hierarchical pattern. The mass matrices, the objects of the theoretical description, can be expressed in terms of powers of a small parameter with coefficients of order one [1]. Since among the quarks the top quark plays an outstanding role, a natural choice for the small parameter is the quantity $\sigma = (m_c/m_t)^{1/2}$ [2]. With this choice the mass ratios $m_u : m_c : m_t$ taken at a common scale are simply $\sigma^4 : \sigma^2 : 1$. Recent indications for solar [3] and atmospheric [4] neutrino oscillations, and thus for non-zero neutrino masses, raise the question of the structure of the mass matrices for leptons. Are they related to the mass matrices of quarks?

In this article I argue for an intimate connection between quark and lepton masses and mixings. The general suggestions from grand unified theories are used: the see-saw mechanism and the expected near equality of the Dirac-neutrino mass matrix with the up-quark mass matrix. For the mass matrix of the heavy neutrinos a new hypothesis is needed. I assume that each heavy neutrino carries a quantum number of a new $U(1)$ symmetry which governs the leading powers of the small parameter σ occurring in this mass matrix. With the single condition that these "charges" are not zero one gets strong restrictions for the form of this mass matrix. The consequences for the light neutrinos are drastic: i) the mass splitting of the two lightest neutrinos is tiny and favors the vacuum oscillation solution for solar neutrinos, and ii) the mixing matrix is close to the one for bimaximal mixing.

2 Quarks

Today the masses and mixings of quarks are reasonably well known with only the exception of the CP-violating phase parameter. After the choice of a convenient basis the mass matrices for up and down quarks can be written down. For instance, one can use as in [2] a real and symmetric matrix m_U for the up quarks and a hermitian matrix m_D for the down quarks such that the elements $(m_U)_{11}$, $(m_D)_{13}, (m_D)_{31}$ and $(m_D)_{23}, (m_D)_{32}$ are strictly zero. This basis has the advantage that the only complex matrix element is $(m_D)_{12} = (m_D)^*_{21}$. More important, by expressing the matrix elements in powers of the small quantity $\sigma = (m_c/m_t)^{1/2} \simeq 0.058 \pm 0.004$ each independent matrix element has a different power of σ for the up as well as the down quark matrices with factors of order 1. Our present knowledge on masses and mixings is compatible with the results obtained from the mass matrices [2] (taken at the mass scale of the vector boson Z):

[2]Compared to ref. [2], the mass of the strange quark is taken to be $m_s \approx \sigma/3\ m_b$ instead of $\approx \sigma/2\ m_b$ and also $m_d/m_b \approx 6\sigma^3$. The smaller values seem to be more appropriate.

$$m_U = \begin{pmatrix} 0 & \sigma^3/\sqrt{2} & \sigma^2 \\ \sigma^3/\sqrt{2} & -\sigma^2/2 & \sigma/\sqrt{2} \\ \sigma^2 & \sigma/\sqrt{2} & 1 \end{pmatrix} m_t, \quad m_D = \begin{pmatrix} 0 & -i\sqrt{2}\sigma^2 & 0 \\ i\sqrt{2}\sigma^2 & -\sigma/3 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_b \qquad (1)$$

and $\sigma = 0.057$. For the purpose of this paper in which we want to use the up quark mass matrix also for neutrinos, it is convenient, however, to transform to a basis in which m_U is diagonal. To leading order in σ one gets

$$m_U = \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad m_D = \begin{pmatrix} O(\sigma^3) & -i\sqrt{2}\sigma^2 & i\sigma^2 \\ i\sqrt{2}\sigma^2 & -\sigma/3 & i\sigma/\sqrt{2} \\ -i\sigma^2 & -i\sigma/\sqrt{2} & 1 \end{pmatrix} m_b \quad . \qquad (2)$$

Clearly, the simple factors in front of the powers of σ in (1) are guessses and have to be changed, or higher order terms in σ have to be included, when more precise information on masses and mixings become available. Also, for definitness, "maximal CP-violation" has been assumed. It is defined to maximize the area of the unitarity triangle with regard to changes of the phases of the off-diagonal elements keeping their magnitudes fixed. Maximum CP-violation defined this way allowed us to bring the off-diagonal elements of m_D into the form of an antisymmetric hermitian matrix [5]. Within the accuracy of only a few degrees one obtains a right-handed unitarity triangle with angles $\alpha \approx 70^o, \quad \beta \approx 20^o, \quad \gamma \approx 90^o$. Independent of the phases the mass matrices (1) demonstrate that masses and mixings are governed by the same small parameter in a simple fashion. With $\sigma = 0.057$ the numerical values for the quark mass ratios at the common scale m_Z and the absolute value of the Cabibbo-Kobayashi-Maskawa matrix CKM as obtained from (1) are

$$\frac{m_u}{m_t} = 1.1 \cdot 10^{-5}, \quad \frac{m_c}{m_t} = 3.2 \cdot 10^{-3}, \quad \frac{m_d}{m_b} = 1.1 \cdot 10^{-3}, \quad \frac{m_s}{m_b} = 2.0 \cdot 10^{-2}, \qquad (3)$$

$$Abs[CKM] = \begin{pmatrix} 0.97 & 0.22 & 0.003 \\ 0.22 & 0.97 & 0.040 \\ 0.010 & 0.039 & 1 \end{pmatrix} . \qquad (4)$$

3 Neutrinos and Charged Leptons

The recent indications for neutrino oscillations imply finite neutrino masses and lepton number violation. For a thorough discussion on possible scenarios and for the relevant literature I refer to ref. [6]. Some approaches based on grand unified theories can be found in [7, 8]. Here, I will take suggestions from grand unified theories without specifications of the group and Higgs representations: The standard model is extended by adding three two-component neutrino fields $\hat{\nu}_e, \hat{\nu}_\mu, \hat{\nu}_\tau$ which are singlets with respect to the standard model gauge group. Since the masses of these fields are not protected, the total 6×6 neutrino mass matrix has a block structure consisting of a 3×3 matrix M with very large entries and a Dirac-type mass matrix m_ν^{Dirac} which connects the light with the heavy fields. At the scale of the heavy neutrinos one expects a close connection between m_{Dirac} and the charged lepton mass matrix m_E with the up-quark mass matrix and the down-quark mass matrix, respectively [5]. For a non-singular matrix M the light neutrinos become Majorana particles according to the see-saw mechanism. Their mass matrix m_ν is given by

$$m_\nu = -m_\nu^{Dirac} \cdot M^{-1} \cdot (m_\nu^{Dirac})^T \quad . \tag{5}$$

In the following I use the relation

$$m_\nu^{Dirac} = m_U \tag{6}$$

and postpone a remark on possible deviations to the end of section 5.

Because the top quark mass is so large compared to all other quark masses, it is convenient to take a basis in which m_U is diagonal as already done in (2). A particular interesting connection between quarks and neutrinos will exist if besides m_U and m_ν^{Dirac} also the mass matrix M has a simple structure in this basis and the parameter σ plays there a similar role. I will explore this possibility.

Let us therefore express the entries of M in terms of powers of σ^2. To give significance to such a form it should be possible to assign $U(1)$ generation charges to the heavy neutrino fields. Generation charges can be decisive for determining the structure of mass matrices, see e.g. [7, 9]. To restrict these charges I will require that the three $U(1)$ quantum numbers differ from each other, are not zero and that not all elements of M vanish in the limit $\sigma \to 0$. As a consequence, two of the three fields must carry opposite charges, and M provides for $\sigma \to 0$ a mass term of the Dirac type for a heavy neutrino, i.e. a neutrino described by two different two-component fields. The mass matrix M which satisfies the requirement and has the entries surviving for $\sigma \to 0$ at the most symmetric place, i.e. $\hat{\nu}_e$ has the opposite charge of $\hat{\nu}_\tau$, has the structure

$$M = \begin{pmatrix} \sim\sigma^6 & \sim\sigma^2 & 1 \\ \sim\sigma^2 & \sim\sigma^2 & \sim\sigma^4 \\ 1 & \sim\sigma^4 & \sim\sigma^6 \end{pmatrix} M_0 \quad . \tag{7}$$

The $U(1)$ charges of $\hat{\nu}_e, \hat{\nu}_\mu, \hat{\nu}_\tau$ are $-3/2$, $1/2$, $3/2$, respectively; they determine the powers of σ^2. If we dismiss matrices which have determinant zero when neglecting higher orders than σ^2, the form (7) is unique apart from a reflection on the cross diagonal corresponding to the charges $-3/2$, $-1/2$, $3/2$. As in the case of the matrix m_U, the unknown factors in (7) should be of order 1 . In particular, if there is a close correlation with m_U, the factor of σ^2 in the first row and first column (p in eq. (8)) should be equal or very close to one. Because of the smallness of σ^4, σ^6 M can be used in the simpler form

$$M = \begin{pmatrix} 0 & p\sigma^2 & 1 \\ p\sigma^2 & r\sigma^2 & 0 \\ 1 & 0 & 0 \end{pmatrix} M_0 \quad . \tag{8}$$

One can check that the approximation (8) is also applicable when calculating m_ν according to (5), (6) even though the inverse of the matrix M enters there. Moreover, a simple consideration of the original 6×6 neutrino mass matrix (with zero entries in the light-light sector) shows that the coefficients p and r can be taken to be real.

For the mass matrix of the light neutrinos, eqs. (5-8) give

$$m_\nu = - \begin{pmatrix} 0 & 0 & r\sigma^2 \\ 0 & 1 & -p \\ r\sigma^2 & -p & p^2 \end{pmatrix} \frac{\sigma^2}{r} \frac{m_t(M_0)^2}{M_0} \quad . \tag{9}$$

The neutrino mass spectrum obtained from this mass matrix is interesting in view of the recent neutrino data. Taking $r = p = 1$ and adjusting M_0 such that the largest eigenvalues (m_3) becomes $m_3 \approx 0.055$ eV, one gets $M_0 \approx 10^{12}$ GeV, $m_3^2 - m_1^2 \approx 3 \cdot 10^{-3}$ (eV)^2 and $m_2^2 - m_1^2 \approx 10^{-11}$(eV)^2 . Furthermore, the neutrino mixing matrix obtained from (9) with $p = 1$ shows the bimaximal mixing discussed in [10]. Thus, the neutrino mass matrix (9) favors large mixing angles for the atmospheric and for the solar neutrinos, and the vacuum oscillation solution [11] for the latter. But the neutrino mass matrix obtained here seems not compatible with the indications for $\hat{\nu}_\mu \to \hat{\nu}_e$ oscillation reported by the LSND collaboration [12].

Before calculating the neutrino properties in more detail, we have to discuss the contributions from the charged lepton mass matrix and from renormalization group effects. The charged lepton mass matrix cannot be expected to be diagonal in the basis used. But it should resemble the down-quark mass matrix shown in (1). Fortunately, because of the small mixing angles, its precise form is not of importance at present. I just take the suggestion for this matrix from ref. [2], transform it to our basis and use, as an example, CP-violating phases in analogy to m_D.

$$m_E = \begin{pmatrix} 0 & -i\sqrt{\frac{3}{2}}\sigma^2 & i\sigma^2 \\ i\sqrt{\frac{3}{2}}\sigma^2 & -\sigma & i\frac{\sigma}{\sqrt{2}} \\ -i\sigma^2 & -i\frac{\sigma}{\sqrt{2}} & 1 \end{pmatrix} m_\tau \quad . \tag{10}$$

By diagonalizing m_E

$$m_E = U_E m_E^{diagonal} U_E^\dagger \tag{11}$$

the neutrino mass matrix (in the basis in which the charged lepton matrix is diagonal) reads

$$\tilde{m}_\nu = U_E^T m_\nu U_E \ . \tag{12}$$

Because U_E is not a real matrix, CP-violation effects are predicted. For CP-conserving processes it will turn out that the influence of U_E on m_ν is not essential, however. Before giving numerical examples, the effects of the scale changes between the high scale M_0 and the weak scale has to be studied.

4 Renormalization group effects

The existence of generation quantum numbers insures the stability of the mass matrices against strong loop corrections. Since the charges of the heavy neutrinos are now fixed one can give corresponding charges to the up-quarks. Because of (6) the singlet anti-up-quark fields may carry the same charges as the heavy neutrinos. The structure of m_U then suggests that the left handed u-quark, charm quark and top quark fields have the charges 7/2 , 1/2 , -3/2 , respectively.

The close connection between quark and lepton mass matrices assumed here must have its origin at the high scale M_0 which, as we have seen, is of order $10^{11}-10^{12}\ GeV$. If not before, at least at this scale new physics will set in. It could modify the scale dependence of the gauge-coupling constant g_1 such as to unify with g_2 and g_3 at their meeting point at $10^{16}\ GeV$. In any case our task is to fix $\tilde{m}_\nu$ at the scale M_0 and to study the behaviour of $\tilde{m}_\nu$ between m_Z and M_0.

When applying the renormalization group equations to the charged leptons, it is of advantage to transform – at all scales relevant here – the right-handed charged leptons such that the corresponding mass matrix contains the left-handed mixing matrix only

$$m_E = U_E\ m_E^{diagonal} \tag{13}$$

where $m_E^{diagonal}$ is a diagonal and positive definite real matrix. By inserting this matrix into the renormalization group equation, one observes that the scale changes concern the mass eigenvalues only. U_E remains invariant: Since below M_0 the masses of the heavy neutrinos do not appear in the renormalization group equation the product $U_E^\dagger \cdot \frac{\partial}{\partial t} U_E$ is a real diagonal matrix. This property suffices to insure that the unitary matrix U_E is independent of the scale function $t = \ln \mu/\mu_0$. Consequently, U_E computed from (10), (11) can also be used at the scale M_0.

At the scale M_0 the mass matrix M for the heavy neutrinos is obtained by replacing in (8) σ^2 by $\sigma^2(M_0) = m_c(M_0)/m_t(M_0)$. The mass matrix m_ν for the light neutrinos becomes [3]

$$m_\nu(M_0) = -\begin{pmatrix} 0 & 0 & r\frac{m_u(M_0)}{m_c(M_0)} \\ 0 & 1 & -p \\ r\frac{m_u(M_0)}{m_c(M_0)} & -p & p^2 \end{pmatrix} \frac{m_c(M_0) m_t(M_0)}{r M_0} \quad . \tag{14}$$

It remains to solve the renormalization group equation for $\tilde{m}_\nu(\mu)$ with the boundary condition

$$\tilde{m}_\nu(M_0) = U_E^T m_\nu(M_0)\ U_E\ . \tag{15}$$

According to ref. [13] one has

$$(4\pi)^2 \frac{d}{dt}\tilde{m}_\nu \quad = \quad (-3g_2^2 + 2\lambda)\ \tilde{m}_\nu + \frac{4}{v^2} Tr(3m_U m_U^\dagger + 3m_D m_D^\dagger + m_E m_E^\dagger)\ \tilde{m}_\nu$$

[3]Because of the uncertainties of the quark masses it is not clear whether the relation $m_u/m_t = (m_c/m_t)^2$ which is not strictly scale-invariant but used in (2) and (6) holds better at m_Z or at M_0. If it holds at M_0, eq. (14) and eq. (9) (with $\sigma = \sigma(M_0)$ and $m_t = m_t(M_0)$) are identical.

$$- \frac{1}{v^2}(\tilde{m}_\nu m_E^\dagger m_E + m_E^T m_E^* \tilde{m}_\nu) \quad . \tag{16}$$

$\lambda = \lambda(t)$ denotes the Higgs coupling constant related to the Higgs mass according to $m_H = \lambda v^2$ with $v = 246\ GeV$. We take $m_H(m_Z) = 140\ GeV$ for the numerical estimates. Eq(16) simplifies since according to (12) and (15) m_E has to be taken in diagonal form. Solving it gives the neutrino mass matrix $\tilde{m}_\nu$ at the scale of the standard model. The neutrino mixing matrix $U = U(m_Z)$ can then be obtained by diagonalizing the hermitian matrix $\tilde{m}_\nu \cdot \tilde{m}_\nu^*$:

$$\tilde{m}_\nu(m_Z) \cdot \tilde{m}_\nu^*(m_Z) = U D D^* U^\dagger \ . \tag{17}$$

The diagonal matrix D

$$D = U^\dagger \tilde{m}_\nu(m_Z)\ U^* \tag{18}$$

gives us the (complex) neutrino mass eigenvalues. By introducing the diagonal phase matrix Φ which consists of the phase factors of D with angles divided by 2, U can be redefined: $U \to \hat{U} = U\Phi$ such that (18) gives now positive definite neutrino mass eigenvalues. $\hat{U}$ expresses the light neutrino states ν_e, ν_μ, ν_τ by the neutrino mass eigenstates ν_1, ν_2, ν_3 according to

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \hat{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad . \tag{19}$$

It turns out that the mixing matrix is not strikingly different from the mixing matrix obtained by diagonalizing (9) , but it contains CP violating phases.

5 Results and discussions

As shown in section 2 the mass matrices of charged fermions have a simple structure. We know much less about the neutrino mass matrix but it is tempting to assume that there exists an intimate relation between the up quark mass matrix and the mass matrix of the heavy neutrinos (the singlets with respect to the standard model gauge group). Because the singlet neutrino fields couple among each other, already the mere existence of a generation quantum number which governs the powers of σ severly restricts the structure of this matrix. Apart from the scale M_0 we are left with essentially only two parameters (r and p). Applying then the see-saw mechanisme we arrived at an interesting mass matrix for the light neutrinos (Eq(9)). The neutrino mass spectrum obtained from it consists of two nearly degenerate states which are lighter by a factor of order σ^2 than the third neutrino. Diagonalization of the neutrino mass matrix gives large mixing angles. Taking the heaviest mass of the light neutrino to be about $5 \cdot 10^{-2} eV$ the mass scale of the singlet neutrinos is of order $10^{12}\ GeV$. Scaling the mass matrix down from this value to the weak interaction scale and including also the mixings of the charged leptons, leads to corrections but does not change the general picture. The charged lepton matrix, together with the neutrino matrix, causes CP violating effects, however. For an illustration the form (10) of the charged lepton mass matrix is used in the following numerical examples.

Let us start by putting the parameter p equal to one. This is an appealing choice because of the corresponding factor one in the up quark mass matrix. With this value the neutrino mixing matrix U as obtained from (17) is of bimaximal type: Almost independent of the parameter r the magnitudes of the elements of the mixing matrix are

$$Abs[U] = \begin{pmatrix} 0.70 & 0.71 & 0.05 \\ 0.50 & 0.50 & 0.71 \\ 0.50 & 0.50 & 0.71 \end{pmatrix} . \tag{20}$$

To obtain contact with the atmospheric neutrino data [4] the product $r \cdot M_0$ can be adjusted to give the heaviest of the light neutrinos a mass of $5.5 \cdot 10^{-2} eV$. One finds $r \cdot M_0 \approx 7 \cdot 10^{11}\ GeV$. The masses of the two lighter neutrinos are then $\approx r \cdot 6 \cdot 10^{-5}\ eV$. The mass splitting between these neutrino depends on the parameter r in a more involved way. One has e.g. $m_2^2 - m_1^2$ equal to $0.8 \cdot 10^{-11}$, $6.5 \cdot 10^{-11}$, $2.2 \cdot 10^{-10}$ and $5.2 \cdot 10^{-10}$ $(\mathrm{eV})^2$ for $r = 1$, $r = 2$, $r = 3$ and $r = 4$, respectivly. These mass differences are in the region of the ones needed for the vacuum oscillation solution for solar neutrinos [11].

To describe the neutrino surviving and transition probabilities it is convenient to introduce the abbreviations

$$S_{ik} = \sin^2(1.27\ (m_i^2 - m_k^2)\frac{L}{E})\ , \quad T_{ik} = \sin(2.54\ (m_i^2 - m_k^2)\frac{L}{E})\ . \tag{21}$$

The mass differences $m_i^2 - m_k^2$ are taken in units of $(eV)^2$, the neutrino energy E in MeV and L, the distance between generation and detection point in meter. The probabilities obtained from (19) for $p = 1$ and $r = 2$ are

$$\begin{aligned} P(\nu_e \to \nu_e) &= 1 - S_{21} - 0.004\ S_{31} - 0.004\ S_{32} \\ P(\nu_\mu \to \nu_\mu) &= 1 - 0.25\ S_{21} - 0.51\ S_{31} - 0.49\ S_{32} \\ P(\nu_\tau \to \nu_\tau) &= 1 - 0.25\ S_{21} - 0.50\ S_{31} - 0.50\ S_{32} \\ P(\nu_e \to \nu_\mu) &= 0.50\ S_{21} + 0.007\ S_{31} - 0.003\ S_{32} \\ &+ 0.02\ T_{21} - 0.02\ T_{31} + 0.02\ T_{32} \\ P(\nu_e \to \nu_\tau) &= 0.50\ S_{21} - 0.003\ S_{31} + 0.007\ S_{32} \\ &- 0.02\ T_{21} + 0.02\ T_{31} - 0.02\ T_{32} \\ P(\nu_\mu \to \nu_\tau) &= -0.25\ S_{21} + 0.50\ S_{31} + 0.49\ S_{32} \\ &+ 0.02\ T_{21} - 0.02\ T_{31} + 0.02\ T_{32}\ . \end{aligned} \tag{22}$$

Only the small numbers appearing in (22) depend notably on the value of the parameter r.

For the solar neutrinos one can set $S_{31} = S_{32} = 1/2$, $T_{31} = T_{32} = 0$. For the atmospheric neutrinos, on the other hand, one can put $S_{21} = T_{21} = 0$, $S_{31} =$

S_{32}, $T_{31} = T_{32}$. From (22) maximal mixing for the solar as well as the atmospheric neutrinos is obvious. It is also seen, that CP violating effects described by the factors multipying T_{ik} are small in this scenario.

The bimaximal mixing obtained so far gets spoiled if the parameter p is sizeable different from one: the mixing angle relevant for atmospheric neutrinos is sensitive to the value of p. Still, deviations from $p = 1$ by up to 25 % are tolerable. $p = 0.75$ e.g. gives for $P(\nu_\mu \to \nu_\mu)$ and $S_{21} = 0,\ S_{31} = S_{32}$

$$P(\nu_\mu \to \nu_\mu) = 1 - 0.93\ S_{31}\ . \tag{23}$$

The Dirac neutrino matrix may differ from the up quark mass matrix. However, if both matrices commute at the scale of M_0, as one might expect, the corresponding changes can be absorbed into the parameters r and M_0. An effective parameter $r \approx 10$ would lead to a mass difference $m_2^2 - m_1^2 \approx 10^{-8}\ (eV)^2$ and thus to an energy independent suppression of solar neutrinos, in some conflict with the results of the Homestake collaboration.

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